Mathematics

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"The use of manipulatives and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts."
– Western and Northern Canadian Protocol in *The Common Curriculum Framework for K–9 Mathematics*

The move toward differentiated instruction in the mathematics classroom represents a significant shift in the way teachers think about and teach mathematics. Today's classrooms are becoming more student-centred, focused less on direct instruction and more on experiential learning. Creating a differentiated mathematics classroom can begin with the following steps (Bender 2005).

- **Consider how students learn.** All of the instructional and assessment decisions you make should be based on the learning needs and preferences of your students.
- **Organize students into flexible groups.** In a differentiated classroom, students are subdivided into groups early and often. There also are multiple opportunities to work both with partners and independently.
- **Plan for a variety of activities.** Learners benefit when content is presented in multiple ways.
- **Connect to the students' world.** Connecting mathematics learning with reference points in the students'community is one way to increase engagement.
- **Create a safe and caring environment.** Create a learning environment in which students feel confident in taking risks and trying new things.
- **Begin differentiated instruction slowly.** Start with an area of mathematics you feel confident teaching. This increases the chances of success and will help create confidence to extend differentiated instruction into other areas.

Use flexible groupings

Effective flexible grouping depends on a thorough knowledge of the curriculum and of individual students in the classroom. Using this knowledge, you can plan a variety of group structures to help students learn mathematics and interact with others in useful ways.

Groups should be formed in a number of ways to give students a variety of learning experiences. For some activities, grouping based on readiness may be the most practical method. Groups also can be formed on the basis of students' general areas of mathematical strengths and challenges, learning preferences or interests. A variety of information, including ongoing assessment results, portfolios, grades and report card comments from previous years, and other

achievement information, can be helpful in identifying groups of students who may need extra assistance and support throughout the year. Flexible grouping can increase students' engagement and success when they have learning opportunities that build on their strengths and needs.

Readiness grouping

Grouping students based on their readiness related to specific concepts or outcomes can allow you to effectively meet the needs of students at multiple levels. The "guess, assess and break away" strategy is one approach to readiness grouping. Using this strategy, you:

- *guess* which students have the concept
- *assess* those several students with one or two quick activities such as a journal prompt or a hands-on activity
- *break away* a small instructional group of those students who will benefit from a targetted instructional activity.

For example, teachers can identify and work with a small break-away group of students doing a follow-up instructional activity after each phase in a learning activity sequence. The break-away group could include students who require additional practice with the concept.

You can use break-away activities throughout a teaching plan. Structure learning activities so that different groups of students participate in the same activity at different times. Individual students might benefit from participating in the same activity with different groups of peers. The different groupings will provide increased opportunities for practice and improved opportunities for engagement. In addition, consider providing break-away activities that incorporate various multiple intelligences.

Use an assessment task prior to beginning a unit. Forming two short-term groups based on the information gathered is a good starting point for differentiating instruction. Similarly, a "do now" or warm-up activity at the start of a learning activity can be used to identify students who need to review a concept before moving on and those who are ready to move forward.

In addition to identifying student readiness levels, assessment tasks can be used to help determine student understanding on the continuum of concrete to abstract. This type of information provides insight into the way some students may need to begin processing the content.

Effective assessment in mathematics requires teachers to plan regular and specific ways to find out *what* students understand and *why* they may be making errors. The knowledge gained through this type of assessment lets teachers differentiate instruction to both assist and challenge students appropriately.

When working with diagrams and manipulatives, mixed-ability grouping may make sense; e.g., the student who works best with manipulatives is working alongside another student who prefers to work symbolically. The goal is that all students will eventually move to the symbolic.

Learning preference grouping

Learning preferences can be the basis of grouping for exploring and applying new mathematical concepts. Consider the following Grade 7 learner outcome related to patterns: Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.

While learners with auditory strengths may be able to describe relationships demonstrated, learners who prefer visual opportunities will benefit from creating graphs, either with paper and pencil or with a software program.

Using tape to create a large graph on the floor will allow the kinesthetic learners in the class to actually become points on the graph.

In this learning activity, students are offered different ways to process the learning, yet the content is the same for every student. Students need opportunities to explore, practise and demonstrate their learning in a variety of ways.

Interest grouping

Students can be in groups based on their interests as well. In this example, all students reach the same goal, but through a choice of three different scenarios.

> Consider the following Grade 2 learner outcome:
> Formulate the questions and categories for data collection, and actively collect first-hand information.
> In this activity:
> - Group 1 is going to investigate something about student eating habits
> - Group 2 is going to investigate something about student hobbies.
> - Group 3 is going to investigate something about student families.
>
> All groups consider the following same questions before constructing their graph.
> - Will your group gather information by counting, measuring or surveying?
> - How do you plan to keep a record of the information gathered?
> - Will you make a bar graph or a pictograph?
> - How will you sort and graph your data?

The simple act of including choice will increase student engagement, particularly if the scenarios for choice are developed based on information about student interests.

Cooperative grouping[1]

Students will benefit from opportunities to participate collaboratively in pairs or in small groups. This approach:

- involves students in working collaboratively to solve a problem or investigate a mathematical idea
- provides opportunities for students to learn from one another
- encourages discussion and sharing of ideas.

Students learn from one another with guidance from the teacher. Shared mathematics promotes the development of problem solving, reasoning and communication skills.

Independent mathematics[2]

Students also need opportunities to work independently to focus on and consolidate their own understanding. Independent mathematics provides opportunities for students to:

- develop, consolidate or apply strategies or skills on their own
- make choices independently
- work at their own pace and develop independence, perseverance and self-confidence
- demonstrate what they know and can do.

Although students are working independently, allow them to ask their peers or you for clarification or feedback. During this time, take the opportunity to observe, ask questions and record information about student understanding, strategies, procedures, skills and knowledge.

Plan differentiatiated learning experiences

Successful planning for differentiation in the mathematics classroom requires teachers to:

- consider the various dimensions of mathematical learning; e.g., computation, explanation, application and problem solving throughout instruction planning
- help students identify their own learning preferences, strengths and challenges, and where they need to grow
- use a variety of teaching strategies to explore a mathematical topic.

1. This section adapted from Expert Panel on Literacy and Numeracy Instruction for Students with Special Education Needs, *Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students with Special Education Needs, Kindergarten to Grade 6* (Toronto, ON: Ontario Ministry of Education, 2005), p. 81. © Queen's Printer for Ontario, 2005. Adapted with permission.
2. Ibid., p. 84.

Plan for a variety of activities

Most students benefit from explicit and focused instruction in targeted chunks of five to 15 minutes. In addition, learners benefit most when content is presented in multiple ways. Present new mathematical concepts at three levels:

- concrete; e.g., using manipulatives such as blocks or other objects
- representational; e.g., pictures
- symbolic.

When planning a learning activity, build in different ways to explore and apply the specific skill or concept, as a whole class or in various grouping configurations. Manipulatives can be used across the grade levels to teach and explore mathematical concepts. Some students find it helpful to explore and visualize problems using manipulatives, and then explain the result to each other. Other students benefit from developing a representation of the problem, while other students learn best by considering it abstractly. The goal should be that students can each demonstrate learning in a variety of ways.

Many teaching resources offer additional activities under the heading of "enrichment" or "alternative teaching" ideas. These activities can serve as a starting point for differentiating instruction.

Limit number of tasks

Generally, students with difficulties in mathematics will achieve greater mastery by focusing on quality (versus quantity) and working with key concepts for longer periods of time.

Search for and emphasize patterns

The brain searches for patterns in order to reduce the information load on the system; therefore, teach using patterns. Present different types of strategies that use patterns (for example, using the patterns of counting by 3s or 5s to help with multiplication) and post these on walls as sample strategies. Use patterns as classroom games or quick and fun activities at various points throughout the school day.

Connect to the students' world

Make connections between students' prior knowledge and new concepts by using real-world examples. Connecting mathematical learning with reference points in the community is one way to increase engagement. Start with the classroom community and move out to include the school, and then the local community. Using local examples can make learning more authentic for students. For example, when learning about mass, students can write word problems with a local flavour; e.g., comparing the mass of various animals at home and then at the local zoo or nearby farm.

Scaffolding instruction

In scaffolded instruction, teachers use individual prompting and guidance tailored to the specific needs of individual students. This provides just enough support for the student to do a new task. Scaffolding can include an array of learning supports, including charts, graphics and cue cards.

For effective scaffolding in the mathematics classroom consider the following.

Identify what students know.
The level of support must be tailored to the students' ever-changing understanding of mathematical concepts. To do this, you need to know what students already know (background or prior knowledge), what misconceptions they have and their developmental level (i.e., which competencies are developing and which are beyond students' current level of functioning). For example, a teacher who knew that her students "think in terms of money," used the familiar concept of money to teach rounding.

Help students achieve successes quickly. If copying numbers is laborious for some students, consider pairing students who have difficulty with written production with a peer buddy who records the answers. This allows the other student to generate ideas on solving problems without having to worry about how to convey these ideas in writing.

Help students learn from each other. Create opportunities for students to teach each other, to summarize main points, and to tutor each other in new concepts in short (e.g., five-minute) tutoring sessions. These types of collaborative activities can enhance student motivation to learn mathematics as well as create opportunities to learn about working with others.

Help students be independent. Effective scaffolding involves listening and watching for clues from students as to when teacher assistance is or is not needed. Achieving independence is different for individual students. Some students may be at identical skill levels, but emotionally they may be at different levels regarding the amount of frustration they can tolerate. Some students will need more teacher support while learning to perform a task; others will demonstrate task mastery more quickly.

Consider the following examples of scaffolding that create choice and provide structured opportunities to reflect on mathematical learning.

Open-ended problem solving

Learning through problem solving needs to be the focus of mathematics at all grade levels. In a problem-solving classroom, problems are used as a vehicle to learn new mathematics.

One way to create this is to use open-ended tasks that provide multiple entry points for students at varying readiness levels. For example,

> The answer is 12, what is the question?

Broken calculator activities are also open-ended. For example,

> Suppose the only working keys are [3] [8] [x] [–] [=].
>
> - Show how you can still get all the answers from 1 to 10.
> - What is the least number of key strokes (button presses) it takes to get each one?

Multiple entry points

John Van de Walle (2001) introduced an interesting model for creating differentiated tasks. By providing multiple sets of data within one problem, students have multiple entry points.

> Maria has {12, 60, 121} pine cones. She gave Evan {5, 15, 46} pine cones. How many pine cones does Maria have now?

Students can complete the task with numbers that are within their reach. For example, a student who finds subtraction challenging can choose 12 – 5, while a student ready for a challenge can choose 121 – 46.

> How many people can you serve with $\{4,\ 4\frac{1}{2}, 2\frac{1}{4}\}$ pizzas, if each person eats $\{\frac{1}{2}, \frac{3}{4}, \frac{2}{3}\}$ of a pizza?

Give students a choice of which pair of numbers to work with first, and then encourage them to continue with a more challenging pair of numbers. This type of task differentiates by readiness, yet engages all students in solving word problems involving division of fractions.

Think-alouds

Modelling how to use "think-alouds" is particularly valuable for students who have difficulty knowing how to select and use appropriate strategies. As the teacher thinks aloud while working through a problem, students hear the thought process involved and see a model of problem-solving behaviour. Think-alouds can be used with the whole class and/or with individual students who need scaffolding support for applying problem-solving strategies.

Journals

Mathematics journals are an opportunity for students to demonstrate and reflect on their understanding of new mathematical concepts. They also provide an opportunity for teachers to learn more about student attitudes toward mathematics. Information from journal entries can be a starting point for grouping students based on:

- levels of understanding
- plans for gaining understanding
- real-world connections and personal interests.

Consider the following types of journal prompts.

- Explain how to solve this problem.
- Identify the error in this solution and explain how you know.
- Compare integers and fractions. How are they the same? How are they different?
- In yesterday's class we learned _____. This concept is important because _____.
- Write down everything you know about triangles.
- Reflect on how you did on the unit test. How did you prepare? How can you improve for next time?

Supporting success for all students

A variety of strategies support a diversity of learning needs in the differentiated mathematics classroom. Many of these strategies also can be used in other subject areas, depending on the content being learned and the level of support required by individual students.

In addition to the strategies taught by the teacher, students may have developed their own personal strategies, or may have learned strategies in other subject areas that are successful for them. Make opportunities during mathematics class to share strategies and encourage students to use their personal strategies in a variety of learning tasks.

Teach how to use textbooks effectively

- Teach text features and how to use them; e.g., bold to identify important ideas or vocabulary, special boxes to set information apart from the rest of the text.
- Teach students to highlight important information found in directions and word problems. Assign different colours for highlighting different parts of the problem (e.g., the question being asked) and have these remain consistent.

Promote comprehension

- Create opportunities for students to use visual images or concrete objects and manipulatives to represent what they have learned. For example, students can visually demonstrate an understanding of equality using a balance scale and 3-dimensional objects.
- Work collaboratively with students to create concept maps to visually represent and demonstrate specific mathematical concepts and/or personal strategies.
- Use word walls and other opportunities to reinforce and create a deeper understanding of new mathematical terms.
- Provide clear directions both verbally and visually.
- Present information in a variety of ways to help students see the information in more than one way. This will help them move the new information into their long-term memory.
- Provide variety and multiple opportunities for participation during learning activities.
- Provide immediate specific feedback for correct and incorrect responses.
- Make learning personally relevant by connecting it to student lives.
- Encourage students to talk aloud while working through a problem or calculation. This can help them organize their thoughts and remember the steps.
- Provide information in small chunks.
- Make connections between each new learning and students' prior knowledge and personal experience.
- Provide multiple opportunities for practise to ensure new learning is embedded in long-term memory.
- Use games for reinforcement and practice. Having fun makes learning more memorable.
- Provide a model of a completed problem and display it so students can refer to it as needed.
- Provide clear and consistent transitions between topics.

Provide supports to reduce frustration and increase success

- Reduce the number of questions on the page. In some cases consider presenting individual problems on single sheets of paper in order to reduce the visual load on students and allow them to focus on one question at a time.
- Provide assistive technology (such as word processors) for students with motor difficulties so they can legibly record their answers.
- Reduce the amount of copying required by providing copies of notes. Students can then highlight the key points on the copies as a way of engaging with the information.
- Reduce the number of assigned mathematical tasks by allowing students a choice; e.g., do any 10.
- Provide extra time to work on assessments tasks, on an as-needed basis.
- Encourage students to use a cue card or post-it note to keep their spot on the page or in the textbook.
- Encourage students to talk aloud while working through a problem or calculation to assist in their organization process and to provide insight into the approach being used.
- Establish a cue (e.g., a phrase or a distinct sound such as a chime) that alerts students that directions are about to be given or instruction is about to begin.

Use mathematics to develop oral, written and reading comprehension skills[3]

- Provide tasks that are worth talking and writing about.
- Model think-aloud techniques and encourage students to do the same; e.g., "I have 25 and need to subtract 7 but don't have enough ones so I need to regroup."
- Model the use of mathematical language.
- Ask good questions and encourage students to reflect on their thinking and ask their own questions.
- Ask, "How do you know?" and create opportunities for students to reflect on their own learning.

The following charts are sample strategies that promote different types of communication including:

- oral communication
- written communication
- reading comprehension.

3. This section and following charts on pages 224–229 adapted from Expert Panel on Literacy and Numeracy Instruction for Students with Special Education Needs, *Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students With Special Education Needs, Kindergarten to Grade 6* (Toronto, ON: Ontario Ministry of Education, 2005), pp. 85–90. © Queen's Printer for Ontario, 2005. Adapted with permission.

Oral communication strategies

Oral communication strategies	Student areas of need that may have an impact on the effectiveness of the strategy	Considerations for implementation
"Think-pair-share"		
Students independently consider a task, strategy, and so on, then pair with another student and share ideas. Two pairs then join each other to discuss further.	• Language abilities—the ability to process information, make connections, and express ideas and solutions. • Prior knowledge and experience—level of content knowledge required to complete the task.	• Consider pairings that support different levels of language ability. Pairings should: – model good language – challenge thinking. • Provide visual prompts or sentence starters to keep pairs on task. • Consider tasks that support different levels of understanding: – pair a capable student with a less capable student to scaffold learning – pair students of similar abilities to consolidate or extend learning.
"Show-and-Tell"		
Students explain the task to one another, build a representation of the solution and then share their work through pictures, words or diagrams.	• Cognitive abilities—being able to represent thinking in concrete ways. • Metacognitive abilities—identifying and selecting appropriate strategies and organizing information • Language abilities—the ability to process information, make connections and express ideas and solutions.	• Provide models of how to describe and show thinking concretely: "I found the total number of buttons by putting them into groups of 10. And then I counted 10, 20, 30, 31, 32, 33." • Provide a checklist that describes the steps in the process. • Provide a template to help organize work. • Provide a framework for showing and telling that might include prompts or sentence starters.

Oral communication strategies	Student areas of need that may have an impact on the effectiveness of the strategy	Considerations for implementation
"Mathematician's chair"		
Students prepare a problem, then share it with the class or small group.	• Language abilities—the ability to process information, make connections and express ideas and solutions. • Self-regulatory abilities—the ability to plan, organize information, to create and solve a problem, and then to share the solution.	• Model what and how to share using think-aloud. • Use a graphic organizer to model the steps needed to solve a problem.

Written communication strategies

Written communication strategies	Student areas of need that may have an impact on the effectiveness of the strategy	Considerations for implementation
"Mind mapping"		
The teacher records ideas about a concept using key words. He or she draws a mind map showing how the ideas are connected.	• This is a good strategy for all students.	• Provide blank or partial templates to help students organize their thinking.
"Think-talk-write"		
The teacher gives students a problem/ question/prompt to think about. Students take turns in small groups to talk about their ideas. Students then write a response.	• Language abilities—the ability to process information, make connections and express ideas and solutions orally, then in writing. • Prior knowledge and experience—level of content knowledge required to complete the task.	• Consider pairings that support different levels of language ability. • Model mathematical language. • Challenge thinking by asking "How do you know?" • Utilize visual prompts to keep pairs on task. • Provide written prompts or sentence starters for sharing. • Consider pairings that support different levels of understanding: – pair a capable student with a less capable student to scaffold learning – pair students of similar abilities to consolidate or extend learning.

Written communication strategies	Student areas of need that may have an impact on the effectiveness of the strategy	Considerations for implementation
"Place mat"		
Students work in groups of four. Each student records responses in one quadrant of a large sheet of paper. A summary of all responses is written in the centre of the paper.	• Prior knowledge and experience—level of content knowledge required to participate in the task.	• Consider groupings that support different levels of understanding: – group capable students with less capable students to scaffold learning – group students of similar abilities to consolidate or extend learning.
"Graphic organizers"		
Students use Venn diagrams, flowcharts, and T-charts to arrange information visually.	• Self-regulation—the ability to know when to use an organizer, how to use it and how to evaluate its effectiveness.	• Model the appropriate use of different types of organizers. • Provide examples of different forms.
"Mathematics word wall"* and *"Mathematics strategy wall"		
Students refer to mathematics vocabulary and sample problem-solving strategies posted in the classroom while making oral and written responses.	• Working memory—the ability to keep in mind the words and strategies needed while completing a writing task.	• Review vocabulary and strategies often. • Provide examples of how the words and strategies are used. • Colour-code, classify or group words and strategies for easier reference.
"Journals"* and *"Learning Logs"		
Students represent their understanding of mathematical concepts by contributing responses, explanations and reflections, using pictures, numbers and/or words.	• Self-regulation—the ability to organize what has just been experienced and then provide a recording of it. • Working memory—the ability to hold in mind what to write, the grammar needed to write and the style to use.	• Provide sentence starters, outlines and models. • Model the use of pictures and diagrams. • Teach the writing form using different examples and contexts. • Provide a checklist of the content to be included in the journal/learning log.

Written communication strategies	Student areas of need that may have an impact on the effectiveness of the strategy	Considerations for implementation
"Mathematics picture books"		
Students write and illustrate a picture book individually, in pairs or as a whole class to explain a concept.	• Self-regulation—the ability to plan and organize an entire story.	• Provide a model using published picture books. • Provide think sheets for planning.
"Poster projects"		
Concepts are represented in poster form using pictures, diagrams and written explanations.	• Self-regulation—the ability to plan and organize a poster.	• Provide examples; use picture supports.
"Students' problem posing"		
Students write their own problems and share them with the class.	• Language abilities—the ability to process information, make connections and express ideas and solutions in writing, then orally.	• Provide a checklist for students to use to write their own problems. • Model what and how to share, using think-aloud.

Reading comprehension strategies

Reading comprehension strategies	Student areas of need that may have an impact on the effectiveness of the strategy	Considerations for implementation
"Retell, Reflect, Relate"		
Students answer questions before, during and after the reading of the problem or task.	• Language abilities—the ability to make connections, use vocabulary and express thinking. • Working memory—the ability to hold in mind important information from the text.	• Use alternative forms of presentation (oral: discuss the problem; visual: present the problem in a picture). • Discuss new vocabulary (add to mathematics wall). • Provide students with a graphic organizer to work through the problem.
"Mental Imagery"		
Students try to represent the problem through the use of images.	• Working memory—the ability to hold in mind important information from the text. • Prior knowledge and experience (vocabulary).	• Generate an image/drawing. • Act out the problem. • Present the problem to students using different media (e.g., audio, picture) to respond to varying learning styles.

Create a safe and caring environment

Create a learning environment in which students feel confident in taking risks and trying new things. Ways to do this include asking questions with no wrong answers and explicitly teaching students how to listen to and support one another.

In the differentiated mathematics classroom, as in any classroom, fostering a positive attitude toward learning is important. Create an environment that encourages success in mathematics. Stress the importance of mathematics as a life skill through the use of real-life situations, and incorporate a problem-solving approach to build on student ability to think analytically and creatively.

Reward and highlight student achievements and/or strengths. Set attainable goals with students and monitor progress on a predetermined schedule. Celebrate successes along the way, and encourage students to reinforce themselves for setting and achieving goals. Emphasize the effort that went into achieving the goals. Help them understand that mistakes help us learn and that mistakes point the way to success. Finally, create an atmosphere of cooperation in which all students are active learners who support each other throughout the learning process.